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# DEMONSTRATION OF A PROPOSITION.

BY THE EDITOR.

PROPOSITION. — *The locus of the vertex of a Parabola which rolls on an equal parabola is a Cissoid.*

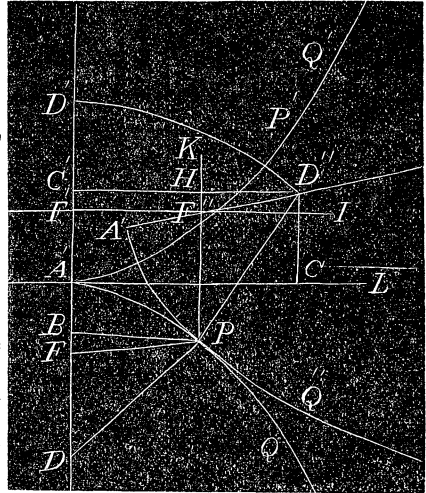
The above proposition is announced, it is said, by Prof. Olney in his General Geometry and Calculus, but without demonstration.

It is proposed here to determine the locus of *any* point in the axis of the rolling parabola, from which the locus of the *vertex* will be found as a particular case of the more general proposition.

Let  $AL$  represent the common vertical tangent to the two parabolas  $APQ$  and  $AP'Q'$ , and let  $F$  and  $F'$  be their foci, and  $F'I$  a directrix through  $F'$ . Let  $P$  be any point in the parabola  $APQ$ , and  $AB$ ,  $BP$  coordinates to the point  $P$ . Then, putting  $AF = a$ ,  $AB = x$  and  $PB = y$  we have, from the property of the parabola,

$$y = 2\sqrt{(ax)}. \dots (1)$$

Suppose the parabola  $AP'Q'$  to have rolled to the position occupied by the parabola  $A'PQ''$ , the two parabolas being now tangent at  $P$ . Draw  $PK$  parallel to the axis  $AB$  and intersecting the directrix in  $F''$ ; then because,



by property of the parabola,  $PF'' = PF$ , and because  $FF''$  is perpendicular to, and is bisected by a tangent at  $P$ , therefore  $F''$  is at the focus of the parabola  $A'PQ''$ ; therefore the focus of the rolling parabola is always on the directrix  $F'I$ , and consequently the locus of  $F'$  is a straight line.

Let  $D'$  represent any point in the axis of the parabola  $AP'Q'$ , at a distance  $b$  from the focus  $F'$ , and let the curve  $D'D''$  represent the locus of  $D'$ ; then,  $A$  being the origin of coordinates,  $AC$  and  $CD''$  will be rectangular coordinates to the point  $D''$ . Put  $AC' = CD'' = x'$  and  $AC = C'D'' = y'$ .

Take  $FD = F'D' (= b)$  and join  $DP$  and  $PD''$ . Then because in the triangles  $DPF$  and  $D''PF''$  the angles at  $D$  and  $D''$  are equal; and because their containing sides  $DF$  and  $DP$ , and  $D''F''$  and  $D''P$  are equal,

each to each, and their opposite sides  $FP$  and  $F''P$  are also equal, therefore the triangles are identical; hence the exterior angles  $PFB$  and  $D''F''H$  are equal. Consequently the triangles  $PFB$  and  $D''F''H$  are similar. Hence

$$FB : BP :: F''H : HD'', \text{ or } (a - x) : y :: F''H : HD''.$$

But because  $DP = D''P$  and  $DF = D''F''$ , we have, (Eucl. 12, II),

$$2FD \times FB = 2F''P \times F''H,$$

$$\text{or} \quad 2b(a - x) = 2(a + x)F''H;$$

$$\therefore \quad F''H = \frac{b(a - x)}{a + x}. \quad \dots \dots \dots (2)$$

Substituting this value for  $F''H$  in the above proportion we have

$$a - x : y :: \frac{b(a - x)}{a + x} : HD'';$$

$$\therefore \quad HD'' = y \frac{b}{a + x}. \quad \dots \dots \dots (3)$$

Hence we have

$$x' = F''H + F'A = \frac{b(a - x)}{a + x} + a \quad \dots \dots \dots (4)$$

$$\text{and} \quad y' = HD'' + BP = y \frac{b}{a + x} + y. \quad \dots \dots \dots (5)$$

From (4) we get

$$x = \frac{a(b + a - x')}{b - a + x'}, \quad \dots \dots \dots (6)$$

and from (5) and (1),

$$y' = 2\sqrt{ax} \left( \frac{b}{a + x} + 1 \right),$$

or, by substituting for  $x$  from (6),

$$y' = (a + b + x') \sqrt{\frac{b + a - x'}{b - a + x'}}, \quad \dots \dots \dots (7)$$

which is the equation to the locus of  $D'$ .

If we take  $b = -a$ ,  $D'$  will coincide with  $A$ , and we shall have for the locus of the vertex

$$y' = \sqrt{\frac{-x'^3}{x' - 2a}} = \sqrt{\frac{x'^3}{2a - x'}},$$

which is the equation of a cissoid.

*Suggestion.*—The circumstance that the locus of  $F'$  is a straight line suggests various applications of the locus of  $D'$  in practical mechanics; as by shifting the point  $D'$  we can produce any desired curvature between a straight line and a curve approximating nearly to that of a circle.